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The propagation constant of the Beverage aerial

No. 1971/39

Research Department, Engineering Division
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RESEARCH DEPARTMENT

THE PROPAGATION CONSTANT OF THE BEVERAGE AERIAL

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October 1971

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THE PROPAGATION CONSTANT OF THE BEVERAGE AERIAL

Summary

The propagation constant for the current flowing in a Beverage aerial driven by a generator is derived from the Compensation Theorem. Comparison is made with measurements and with other theoretical methods. It is concluded that the attenuation increases approximately as the square root of the frequency, reaching a limiting value at high frequencies. The velocity of propagation is less than the free-space velocity but tends towards the latter at high frequencies. Expressions for the characteristic impedance of the aerial are also given.

List of principal symbols

c	velocity of propagation in free space
E	electric field strength
f	frequency, MHz
H	magnetic field strength
h	height of conductor above ground
I	current flowing on the conductor
K	contribution to the surface integral, per unit length along the direction of the conductor
$K_r = \epsilon_r - j1{\cdot}8 \times 10^4 \sigma/f$,	complex relative permittivity of the ground
l	distance between terminals A and B

v	velocity of propagation along the conductor
W	a complex factor defined in Section 2.2
Z_{AB}	mutual impedance between terminals A and B
Z_c	characteristic impedance
α	attenuation constant
β	phase constant
$\gamma = \alpha + j\beta$,	propagation constant
ϵ_r	dielectric constant of the ground
η	intrinsic impedance
λ	wavelength
σ	ground conductivity

Unprimed quantities (e.g. E_z) denote fields, currents etc. for the conductor when over perfectly-conducting ground. Primed quantities (e.g. E'_z) denote the corresponding quantities when the ground is imperfectly conducting. The subscript o (e.g. η_o) is used to denote the intrinsic impedance and propagation constant of free space. Corresponding quantities without subscripts refer to the lower medium.

Rationalised MKS units are used throughout, but the frequency is stated in MHz.

1. Introduction

The Beverage aerial consists of a long wire supported on poles a few metres above the ground, terminated at its far end by a resistor. It was first used in America in 1922 for the reception of European v.l.f. transmitters, and aerials up to 12 km long were constructed.[1] It is currently used by the BBC for the reception of distant m.f. and h.f. transmitters. The use of the aerial for transmission has also been considered.

Whether the aerial is used for transmission or reception, a knowledge of the propagation constant for the current which would travel along it if it were driven is essential if its performance is to be assessed. The propagation constant is derived here by applying the Compensation Theorem,[2] and the result is compared with measurements and with other theoretical methods. Expressions for the characteristic impedance of the aerial are also derived.

2. Theoretical analysis

Fig. 1 shows a Beverage aerial which is driven at a pair of terminals A and has a second pair at B. The distance to the termination is assumed to be so large that no power is reflected from it when the current is attenuated by imperfectly-conducting ground.

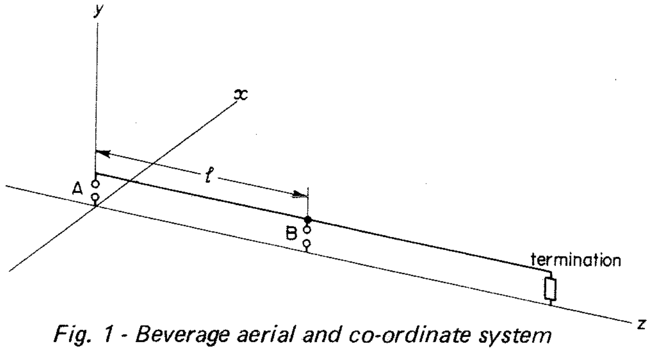

Fig. 1 - Beverage aerial and co-ordinate system

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If the ground were perfectly conducting and the aerial correctly terminated, the mutual impedance Z_{AB} between terminals A and B would be

$$Z_{AB} = Z_c e^{-\gamma_o l} \qquad (1)$$

where Z_c is the characteristic impedance of the conductor over perfectly-conducting ground, γ_o is the free-space propagation constant and l is the distance between A and B. The corresponding expression for imperfectly-conducting ground is

$$Z'_{AB} = Z'_c e^{-\gamma' l} \qquad (2)$$

where Z'_c and γ' are the required values for the characteristic impedance and propagation constant of the aerial.

According to the Compensation Theorem,[2] if a change is made within a closed surface S which excludes two pairs of terminals A and B, the change in mutual impedance is given by

$$Z'_{AB} - Z_{AB} = \frac{1}{I(0)^2}\int_s (\mathbf{E}_A \times \mathbf{H}'_B - \mathbf{E}'_B \times \mathbf{H}_A).\mathrm{d}\mathbf{s} \qquad (3)$$

where

$\mathbf{E}_A, \mathbf{H}_A$ are the electric and magnetic fields over S before the change is made, when a current $I(0)$ is impressed at terminals A

$\mathbf{E}'_B, \mathbf{H}'_B$ are the fields over S after the change, when a current $I(0)$ is impressed at terminals B

ds is an element of S regarded as a vector in the direction of the outward normal from the surface

In the problem considered here, S is the surface of the ground. Fig. 1 shows the co-ordinate system, the origin being situated below the terminals A. The surface S corresponds to the $y = 0$ plane.

Since the ground is initially perfectly conducting, the tangential component of $\mathbf{E}_A$ is zero and the vector product $\mathbf{E}_A \times \mathbf{H}'_B$ therefore makes no contribution to the surface integral. Over perfectly-conducting ground the only component of $\mathbf{H}_A$ which is present is H_x, and it therefore follows that the only component of $\mathbf{E}_B$ which need be considered is E'_z. Consequently Equation (3) simplifies to

$$Z_{AB} - Z'_{AB} = \frac{1}{I(0)^2}\int_s E'_{Bz} H_{Ax}\,\mathrm{d}x\mathrm{d}z \qquad (4)$$

If the currents on the conductor are $I_A(z)$ and $I'_B(z)$ when the aerial is driven at terminals A and B respectively, the contribution to the integral from a strip of ground of width δz extending to infinity on either side of the conductor is proportional to the product of the currents. The contribution from the strip may therefore be expressed in the form

$$\delta z \int_{-\infty}^{\infty} E'_{Bz} H_{Ax}\,\mathrm{d}x = K I_A(z) I'_B(z)\,\delta z \qquad (5)$$

where K is a constant which depends on the height of the conductor and on the ground constants. Equation (5) enables Equation (4) to be simplified to

$$Z_{AB} - Z'_{AB} = \frac{K}{I(0)^2}\int_0^{\infty} I_A(z) I'_B(z)\,\mathrm{d}z \qquad (6)$$

When a current $I(0)$ is impressed at terminals A and the ground is perfectly-conducting, the current distribution $I(z)$ is given by

$$I(z) = I(0) e^{-\gamma_o z} \qquad (7)$$

When a current $I(0)$ is impressed at terminals B it divides equally between the two branches of the aerial and is given by

$$\begin{aligned} I'(z) &= \frac{I(0)}{2}\left[-e^{\gamma'(z-l)} + e^{-\gamma'(z+l)}\right] \qquad 0<z<l \\ &= \frac{I(0)}{2}\left[e^{-\gamma'(z-l)} + e^{-\gamma'(z+l)}\right] \qquad z>l \end{aligned} \qquad (8)$$

The second term on the right-hand side of Equation (8) arises because the current is reflected at $z = 0$, where the terminals A are assumed to be open-circuited.

Substitution of Equations (1), (2), (7) and (8) in Equation (6), followed by integration between the appropriate limits, leads to the following result

$$Z_c e^{-\gamma_o l} - Z'_c e^{-\gamma' l} = K\frac{\gamma' e^{-\gamma' l} - \gamma_o e^{-\gamma_o l}}{\gamma'^2 - \gamma_o^2} \qquad (9)$$

Two special cases, when $l = 0$ and infinity, may now be considered.

(1) As l tends to infinity, the terms containing $\exp(-\gamma' l)$ tend to zero and Equation (9) simplifies to

$$Z_c = \frac{-K\gamma_o}{\gamma'^2 - \gamma_o^2} \qquad (10)$$

Equation (10) gives the following expression for the propagation constant

$$\gamma' = \gamma_o\left[1 - \frac{K}{\gamma_o Z_c}\right]^{1/2} \qquad (11)$$

(2) When $l = 0$, terminals A and B coincide and Equation (9) simplifies to

$$Z_c - Z'_c = \frac{K}{\gamma' + \gamma_o} \qquad (12)$$

Elimination of γ' from Equations (10) and (12) leads to the following expression for the characteristic impedance

$$Z'_c = Z_c\left[1 - \frac{K}{\gamma_o Z_c}\right]^{1/2} \qquad (13)$$

Equations (11) and (13) enable γ' and Z'_c to be determined if K is evaluated. From Equation (5)

$$K = \frac{1}{I(z) I'(z)}\int_{-\infty}^{\infty} E'_z H_x\,\mathrm{d}x \qquad (14)$$

Solutions of this integral are discussed in the sections which follow.

2.1 Approximate solution

When the ground is perfectly conducting the tangential magnetic field H_x is given by

$$H_x = \frac{I(z)h}{\pi(h^2+x^2)} \tag{15}$$

where h is the height of the conductor.

It may be shown that the tangential magnetic field over imperfectly-conducting ground is approximately equal to that over perfectly-conducting ground provided the modulus of the complex relative permittivity K_r is greater than 10. This condition is satisfied by ground of good conductivity at all frequencies and by ground of poor conductivity (10^{-3} S/m) at frequencies below 2 MHz. It follows therefore that E'_z is given approximately by

$$E'_z = -Z_s H'_x = \frac{-Z_s I'(z)h}{\pi(h^2+x^2)} \tag{16}$$

where Z_s is the surface impedance of the ground. Z_s is equal to η, the intrinsic impedance of the lower medium, provided H'_x does not vary over the surface to any great extent within a distance $1/|\gamma|$ where γ is the propagation constant of the lower medium.[2] This condition is assumed to be satisfied for the approximate solution described in this section.

Substitution of Equations (15) and (16) in Equation (14), with Z_s equal to η, leads to the following result

$$K = -\int_{-\infty}^{\infty} \frac{\eta h^2 \,\mathrm{d}x}{\pi^2(h^2+x^2)^2} = \frac{-\eta}{2\pi h} \tag{17}$$

Substitution of this value of K in Equations (11) and (13) then gives the following approximate solutions for γ' and Z'_c

$$\gamma' = \gamma_o \left[1 + \frac{\eta}{2\pi h Z_c \gamma_o}\right]^{\frac{1}{2}} \tag{18}$$

$$Z'_c = Z_c \left[1 + \frac{\eta}{2\pi h Z_c \gamma_o}\right]^{\frac{1}{2}} \tag{19}$$

2.2 More-exact solution

In the approximate solution it was assumed that $E'_z = -\eta H'_x$ and that H'_x is equal to the magnetic field over perfectly-conducting ground. An expression for E'_z which does not require these approximations is derived in this section.

E'_z is given by Maxwell's equation $(\sigma + \mathrm{j}\omega\epsilon)\mathbf{E} = \text{curl } \mathbf{H}$ as follows

$$(\sigma + \mathrm{j}\omega\epsilon)E'_z = \frac{\partial H'_y}{\partial x} - \frac{\partial H'_x}{\partial y} \tag{20}$$

If Equation (20) is applied just below the surface of the ground, the value of E'_z at the surface is obtained.

For perfectly-conducting ground the magnetic fields above the surface are usually calculated by assuming that an image current $-I$ flows at a depth h below the surface. Here it is assumed that the magnetic fields above imperfectly-conducting ground may be calculated in a similar manner, by assuming that the strength of the image current is ρI, where

$$\rho = \frac{1 - K_r^{\frac{1}{2}}}{1 + K_r^{\frac{1}{2}}} \tag{21}$$

and K_r is the complex relative permittivity of the ground.* Although this procedure is somewhat empirical, it yields exact values for the fields when the ground is either perfectly conducting ($K_r = \infty$) or absent ($K_r = 1$); it would therefore be expected to be a reasonable approximation for intermediate values of K_r. It also yields exact values for the fields at a great height above a conductor of finite length, for all values of K_r.

The magnetic fields at the surface calculated in this way are

$$H'_x = \frac{K_r^{\frac{1}{2}} h I'}{\pi(1+K_r^{\frac{1}{2}})(h^2+x^2)} \tag{22}$$

$$H'_y = \frac{x I'}{\pi(1+K_r^{\frac{1}{2}})(h^2+x^2)} \tag{23}$$

The magnetic fields immediately below the surface are identical, since the boundary conditions require **H** to be continuous.

The first term on the right-hand side of Equation (20) may now be obtained by differentiating Equation (23) and is given by

$$\frac{\partial H'_y}{\partial x} = \frac{I'(h^2-x^2)}{\pi(1+K_r^{\frac{1}{2}})(h^2+x^2)^2} = \frac{H'_x(h^2-x^2)}{K_r^{\frac{1}{2}} h (h^2+x^2)} \tag{24}$$

The second term on the right-hand side of Equation (20) may be obtained by assuming that the variation of H'_x below the surface is described by the equation

$$H'_x(y) = H'_x(0)\mathrm{e}^{\gamma_y y} \tag{25}$$

where γ_y is the y-component of the propagation constant of the wave which propagates just below the surface.** In general this wave may be resolved into an angular spectrum of plane waves. Here it will be assumed for convenience that a plane wave having no variation in the x-direction predominates; this assumption is reasonable at high frequencies but liable to fail at low frequencies, where the conductor height is comparable with the penetration depth. For the dominant wave described below

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*ρ is the Fresnel plane-wave reflection coefficient for normally-incident waves

**In Equation (25), the exponent $\gamma_y y$ is positive because power flows into the ground, in the negative-y direction.

$$\gamma_y^2 + \gamma_z^2 = \gamma^2 \qquad (26)$$

where γ_z is the z-component of its propagation constant and γ is the propagation constant for plane waves in the lower medium.

Differentiation of Equation (25) then gives

$$\frac{\partial H'_x}{\partial y} = \gamma_y H'_x \qquad (27)$$

From Equations (22), (26) and (27) it follows that

$$\frac{\partial H'_x}{\partial y} = \frac{-K_r^{1/2} h I'}{\pi(1+K_r^{1/2})(h^2+x^2)} \left[\gamma^2 - \gamma_z^2\right]^{1/2} \qquad (28)$$

Now γ_z is equal to γ', which is to be determined, and $\sigma + j\omega\epsilon = \gamma/\eta$. Using these identities, it may be shown from Equations (20), (24) and (28) that a more-accurate expression for E'_z is

$$E'_z = \eta H'_x \left\{ \frac{h^2 - x^2}{K_r^{1/2}\gamma h(h^2+x^2)} - \left[1 - \left(\frac{\gamma'}{\gamma}\right)^2\right]^{1/2} \right\} \qquad (29)$$

where H'_x is given by Equation (22).

Substitution of Equation (29) in Equation (14), followed by integration, enables K to be expressed in the form

$$K = \frac{-\eta W}{2\pi h} \qquad (30)$$

where

$$W = \frac{-K_r^{1/2}}{1 + K_r^{1/2}} \left\{ \frac{1}{2K_r^{1/2}\gamma h} - \left[1 - \left(\frac{\gamma'}{\gamma}\right)^2\right]^{1/2} \right\} \qquad (31)$$

Comparison of Equations (17) and (30) shows that they are of the same form, and it therefore follows that the more exact-solutions for γ' and Z'_o are given by

$$\gamma' = \gamma_o \left[1 + \frac{\eta W}{2\pi h Z_c \gamma_o}\right]^{1/2} \qquad (32)$$

$$Z'_c = Z_c \left[1 + \frac{\eta W}{2\pi h Z_c \gamma_o}\right]^{1/2} \qquad (33)$$

Since W contains γ', Equation (32) leads to a quadratic equation in γ'^2. Although γ' may be found by solving this equation, it is more convenient to substitute the approximate solution for γ' in the expression for W and then to evaluate γ' and Z'_o directly from Equations (32) and (33). In principle the value of γ' derived from Equation (32) could then be substituted in Equation (31) for a further iteration, but computations have shown that no worthwhile advantage is to be gained from this procedure.

3. Application of the theory

In applying Equations (18) and (19), or (32) and (33) to practical problems, formulae for the following quantities are also required

(1) The characteristic impedance of the conductor over perfectly-conducting ground

$$Z_c = 60 \log (h/a) \qquad (34)$$

where a is the conductor radius

(2) The intrinsic impedance of the ground

$$\eta = \eta_o K_r^{-1/2} \qquad (35)$$

where $\eta_o = 120\pi$ and K_r is the complex relative permittivity, defined as follows

$$K_r = \epsilon_r - j\epsilon'_r \qquad (36)$$

where

ϵ_r	is the dielectric constant of the ground
ϵ'_r	$= 1{\cdot}8 \times 10^4 \sigma/f$
σ	is the ground conductivity in S/m (mhos/m)
f	is the frequency in MHz

(3) The free-space propagation constant

$$\gamma_o = j\beta_o = j2\pi/\lambda_o \qquad (37)$$

where λ_o is the free space wavelength

The more exact solution also requires the value of γ, the propagation constant in the lower medium, which is equal to $\gamma_o K_r^{1/2}$.

In practice the real part a' of the solution for γ' is small. It is more convenient to calculate the attenuation per km, since the latter quantity is usually measured. The attenuation per km is equal to $8686a'$ dB.

It is also more convenient to express the imaginary part β' in terms of the velocity of propagation along the conductor, relative to the free-space velocity (v/c). This is given by

$$\frac{v}{c} = \frac{\beta'}{\beta_o} \qquad (38)$$

and is known as the velocity ratio.

Fig. 2 shows how the velocity ratio and attenuation for the current flowing in a typical aerial* varies with frequency, for a representative range of ground constants. Fig. 2(*a*) shows that the velocity ratio tends to the free-space velocity at high frequencies, and at low frequencies depends on the ground conductivity but is independent of the dielectric constant. Fig. 2(*b*) shows that the ground conductivity also controls the attenuation at low frequencies, but at high frequencies the attenuation tends to a limiting value which depends on the dielectric constant of the ground. Fig. 2(*b*) shows curves for three values of ϵ_r, for σ = 0·003 S/m; curves for other conductivities tend to the same limiting values. The curves of Figs. 2 and 3 were computed by the method described in Section 2.2.

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*The curves of Figs. 2 and 3 apply to aerials having the same dimensions as those erected at Crowsley Park.

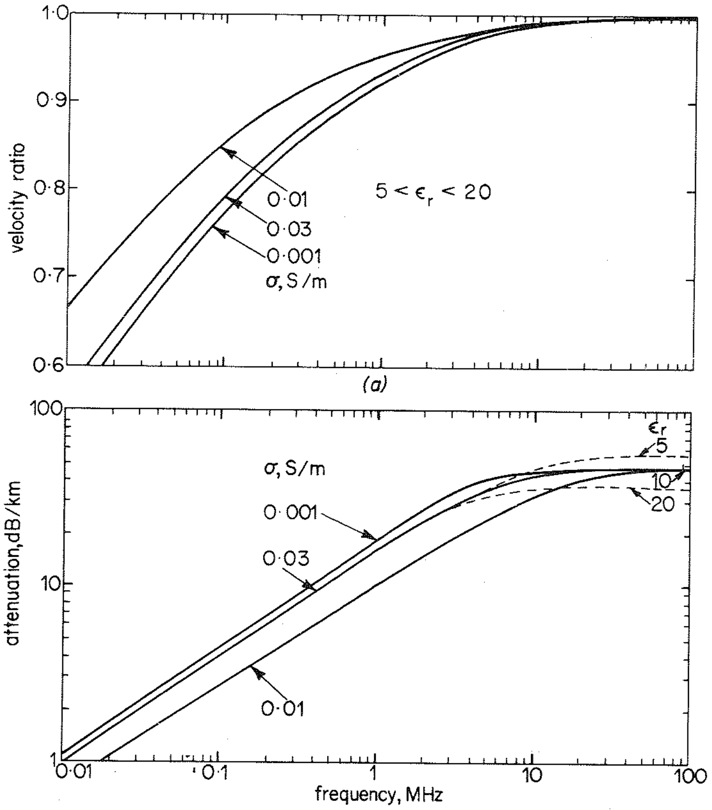

Fig. 2 - Theoretical propagation constant

(a) velocity ratio (b) attenuation

Conductor height, 3 m; perfect-earth characteristic impedance, 426 ohms

Fig. 3 shows the variation of characteristic impedance with frequency. At high frequencies it tends to the perfect-earth value, and at low frequencies it depends only on the ground conductivity.

It may be shown that the variations of propagation constant and characteristic impedance with frequency are exactly the same as those of an air-spaced transmission line of characteristic impedance Z_c, into which a series impedance $Z = \eta W/2\pi h$ per unit length has been inserted. If Z is small compared with $\gamma_0 Z_c$, α' and β' are given approximately by

$$\alpha' \simeq \frac{\mathrm{Re}(Z)}{2Z_c} \tag{39}$$

$$\beta \simeq \beta_0 + \frac{\mathrm{Im}(Z)}{2Z_c} \tag{40}$$

where Re(Z) and Im(Z) denote the real and imaginary parts of Z respectively and $j\beta_0$ is the free-space propagation constant. The characteristic impedance is given approximately by

$$Z'_c \simeq Z_c + \frac{Z}{2\gamma_0} \tag{41}$$

For the approximate solution, $W = 1$ and $Z = \eta/2\pi h$. At h.f., η tends asymptotically to $\eta_0 \epsilon_r^{-1/2}$ and consequently γ' and Z'_c also tend to asymptotic values. Equations (39), (40) and (41) show why the attenuation tends to a value which depends on the dielectric constant, while the velocity ratio and characteristic impedance tend to the perfect-earth values. At low frequencies the real and imaginary parts of η are both approximately equal, and proportional to $f^{1/2}$. At these frequencies, therefore, the attenuation increases as $f^{1/2}$ and the velocity of propagation deviates from the free-space velocity approximately as $f^{-1/2}$.

The trends described above apply also to the more-exact solution. The principal differences between the two solutions arise at h.f., where the factor W differs appreciably from unity, but the only significant effect at h.f. is an appreciable reduction in the attenuation.

Equations (39) and (40) also show that increasing the height of the conductor reduces both the attenuation and the velocity deviation; the effect is augmented by the small increase in Z_c. Raising the height of a Beverage aerial is not necessarily advantageous, however, since greater voltages are induced in the vertical end wires. The use of two or more conductors in parallel offers no advantage, since Z_c is reduced and the attenuation thereby increased.

4. Comparison with measurements

The impedance/frequency characteristic of an 880 m Beverage aerial at Crowsley Park near Reading was

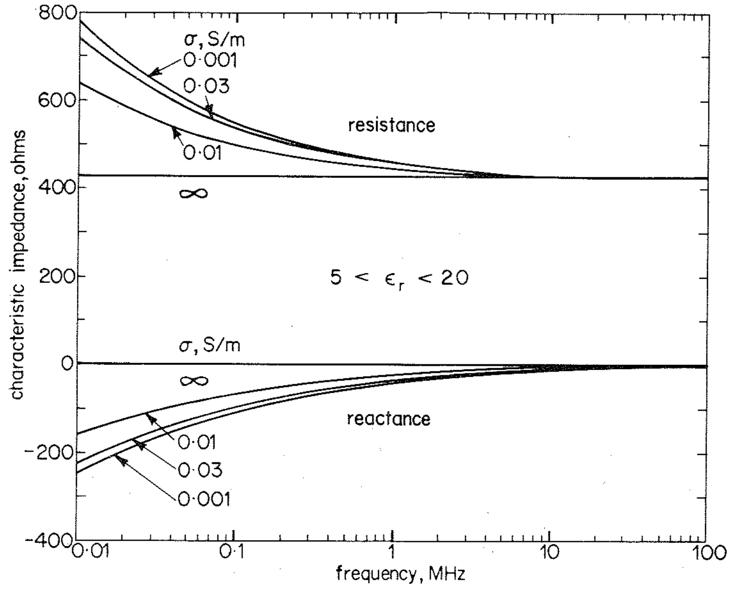

Fig. 3 - Theoretical characteristic impedance

Conductor height, 3 m

Perfect-earth characteristic impedance, 426 ohms measured with the aerial open-circuited at its far end. The aerial consisted of a single conductor of bare 6 SWG (4·88 mm diameter) copper wire erected at a height of 3·0 m above ground.

The velocity ratio was calculated by comparing the measured electrical length with the electrical length which the aerial would have had if v were equal to c. The attenuation was derived from the relative amplitudes of the forward and reflected waves, the reflection coefficient at the far end being assumed to be unity.

In Fig. 4 the measured values are compared with computations, by the method described in Section 2.2, for two ground conductivities; the conductivity at Crowsley Park is believed to lie between these values. The measured velocity ratios agree well with theory, but the measured attenuations are somewhat lower than the theory predicts.

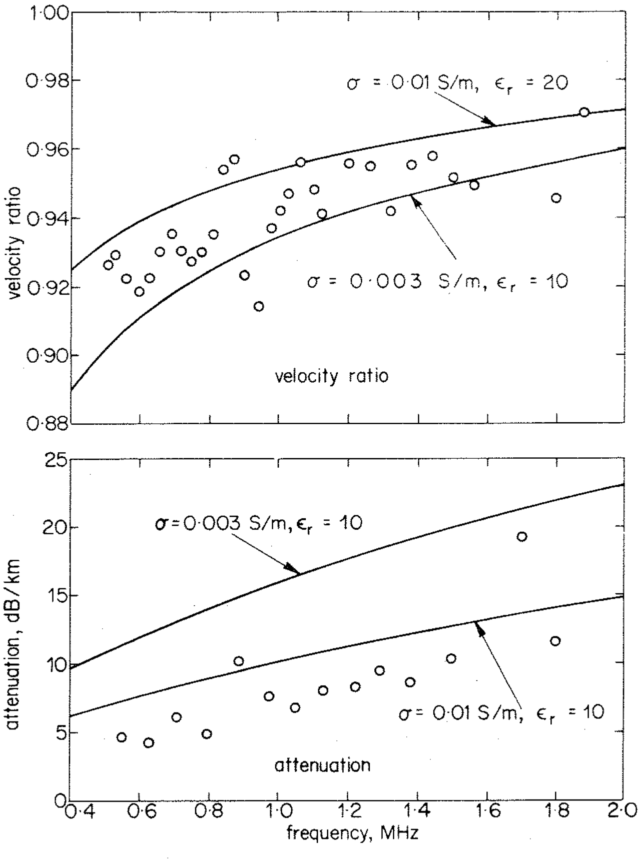

Fig. 4 - Comparison of theoretical and measured propagation constants

Conductor height, 3 m
Perfect-earth characteristic impedance, 426 ohms

Impedance/frequency measurements were also made on a 420 m aerial which was otherwise identical, but terminated in 500 ohms. The measured impedance was approximately equal to the characteristic impedance of the aerial; more precise values of Z'_c were derived from the geometric means of measurements made at pairs of frequencies chosen such that the electrical lengths of the aerial differed by $\lambda/4$. Fig. 5 shows a comparison of values calculated in this way with values computed by the more-exact method; the agreement with theory is seen to be reasonable. The characteristic impedance was also calculated from the measurements on the open-circuited 880 m Beverage and similar results were obtained, but the scatter of the calculated values was somewhat greater because of the greater variation in the measured impedances, due to the aerial being open-circuited.

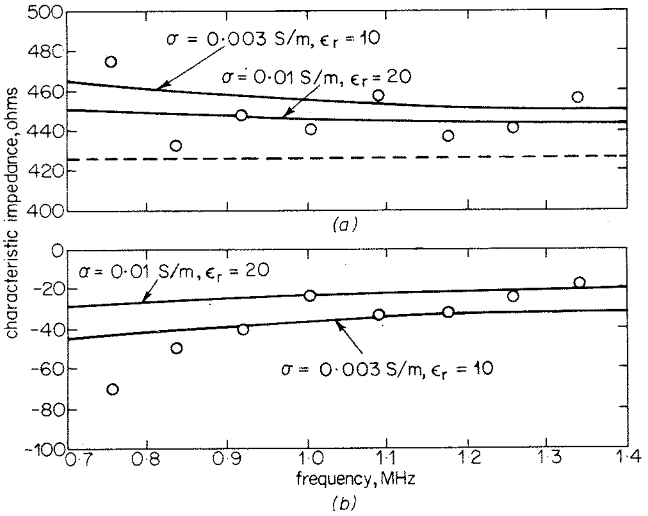

Fig. 5 - Comparison of theoretical and measured characteristic impedance

(a) resistance (b) reactance

—— theoretical

– – – – perfect-earth characteristic impedance

○ measured values

Conductor height 3 m

A comparison has also been made with velocity ratios and attenuations measured at v.l.f. by Beverage et al.[1] One of their aerials consisted of a single 10 AWG (2·59 mm diameter) bare copper conductor erected at a height of 8 m over ground whose conductivity was said to be 5 x 10^{-3} S/m. The measured velocity ratios and attenuations are compared with theoretical values, calculated by the method described in Section 2.2, in Table 1.

Table 1

Comparison of theoretical and measured propagation constants

Frequency, kHz	12	20	30
Theoretical velocity ratio	0·823	0·857	0·880
Measured velocity ratio	0·827	0·840	0·853
Theoretical attenuation, dB/km	0·40	0·54	0·68
Measured attenuation, dB/km	0·20	0·27	0·34

Table 1 shows that the theoretical velocity ratios agree reasonably well with measurements, but the theoretical attenuations are twice as large as the measured values.

Beverage also describes measurements made on 2-wire and 4-wire aerials; these show lower velocity ratios and higher attenuations than those measured with single-wire aerials. This result is consistent with theory.

5. Comparison with other theories

It is of interest to compare the present theory with other methods which have been used to solve the same problem. Of particular interest is the method used by Carson,[3] which is restricted to low frequencies because displacement currents in the ground are neglected.* Carson's method was later extended to higher frequencies by Wise.[5]

Carson expresses the field above the ground in the form of an angular spectrum of plane waves and equates this to the sum of the fields due to the current in the conductor and the conduction current in the ground. This leads to an expression for the series impedance arising from the imperfect conductivity, in the form of a definite integral. Carson's paper contains curves and formulae which enable the integral to be evaluated.

Carson's formulae and curves are expressed in terms of a parameter r, which may be shown to be equal to $2\beta_o h(\epsilon'_r)^{1/2}$ where ϵ'_r is the imaginary part of the complex relative permittivity of the ground, defined in Equation (36). Thus r varies as $f^{-1/2}$ and is small at low frequencies. An interesting feature of Carson's theory is that the real part of the integral tends to a limiting value when r is small. A consequence is that the series resistance is then directly proportional to frequency and the attenuation increases in the same way. This result conflicts with that derived from the Compensation Theorem, which yields attenuations proportional to $f^{1/2}$ at low frequencies. Neither is it confirmed by Beverage's measurements, which also vary as $f^{1/2}$ (as can be seen from Table 1) even though r is small. Attenuations measured by Jenssen[6] at m.f. also varied less rapidly than Carson's theory predicts. Variation as $f^{1/2}$ is to be expected intuitively, because both the complex relative permittivity of the ground, and the penetration depth, vary inversely as $f^{1/2}$.

Wise extended Carson's theory to include the effect of displacement currents, and his theory is not therefore subject to an upper frequency limit. Wise replaced Carson's parameter r by a complex parameter which can be shown to be equal to

$$2\beta_o h \left[j(K_r - 1) \right]^{1/2}$$

At low frequencies, $K_r \gg 1$ and $jK_r \simeq \epsilon'_r$ and the two parameters have similar values; both theories then give identical results. At high frequencies, $K_r \simeq \epsilon_r$ and it may be shown that the series impedance tends asymptotically to $\eta/2\pi h$, where η is the intrinsic impedance of the ground. This result is identical with that given at all frequencies by the Compensation Theorem, using the approximate solution described in Section 2.1.

Reference must also be made to Kikuchi,[7] who also extended Carson's theory to higher frequencies; he does not appear to have been aware of the paper by Wise. Kikuchi's paper contains a curve showing the theoretical attenuation of a conductor 7·5 m above ground of good conductivity (10^{-2} S/m) at frequencies between 1 and 10^4 MHz. The attenuation increases, following Carson's theory, up to 10 MHz and then falls, reaching a minimum value at about 100 MHz. At even higher frequencies it assumes the attenuation appropriate to an axial-cylindrical surface wave[8] supported by a conductor of finite resistance in free space. This result is hardly surprising because Kikuchi assumes the dielectric constant of the ground to be unity, and consequently the ground approximates to free space at frequencies above 1000 MHz. Unfortunately Kikuchi's paper does not appear to contain sufficient information to enable attenuations to be calculated for more practical cases.*

6. Conclusions

Use of the Compensation Theorem has shown that the velocity of propagation of the current flowing on a Beverage aerial when driven approaches the free-space velocity at high frequencies. At low frequencies the velocity is less than the free-space velocity, the reduction in velocity depending on the ground conductivity and on the height of the conductor. These factors also govern the rate of attenuation at low frequencies, where the theory indicates that the attenuation is proportional to the square root of the frequency. At high frequencies, the attenuation increases to a limiting value which depends on the dielectric constant of the ground.

Increasing the height of the conductor reduces the attenuation and makes the velocity of propagation more nearly equal to the free-space velocity. In a long Beverage receiving aerial this would result in an improvement in sensitivity and directivity, provided the greater voltages induced in the vertical end wires can be cancelled. The use of multiple wires reduces the velocity and increases the attenuation, and therefore degrades the performance of a Beverage aerial.

The theory shows that the characteristic impedance of the aerial is capacitive and greater than the characteristic impedance of an identical aerial erected over perfectly-conducting ground. At high frequencies, however, the characteristic impedance tends towards the perfect-earth value.

Theoretical propagation constants show reasonably good agreement with measurements. At m.f. the theoretical velocity of propagation agrees well with measured values, but the attenuation is slightly over-estimated. At v.l.f. the measured velocity still shows good agreement with theory, but the theoretical attenuation is too great, by a factor of about two. Comparison between theory and measurement at h.f. has yet to be made.

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* A similar solution was published independently by Pollaczek.[4]

*It is perhaps worth noting that the more-exact solution described in Section 2.2 also shows a reduction in attenuation at v.h.f. if ϵ_r is specified as unity, although the result obtained differs from Kikuchi's.

The theory based on the Compensation Theorem is relatively easy to apply and appears to give results at m.f. and h.f. which are sufficiently accurate for practical purposes. At h.f. the theory is consistent with the more complicated Carson-Wise theory, but the two theories disagree at v.l.f. Here the Carson-Wise theory predicts attenuations proportional to f, where f is the frequency, while the present theory suggests that they vary at $f^{1/2}$. Although the approximations contained in the theory described in this report are responsible for overestimating the attenuation at v.l.f., the rate at which the attenuation varies with frequency is less liable to error and is, moreover, consistent with measurements.

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8. References

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